Worst-Case Hardness for LPN
(and Cryptographic Hashing)
via Code Smoothing

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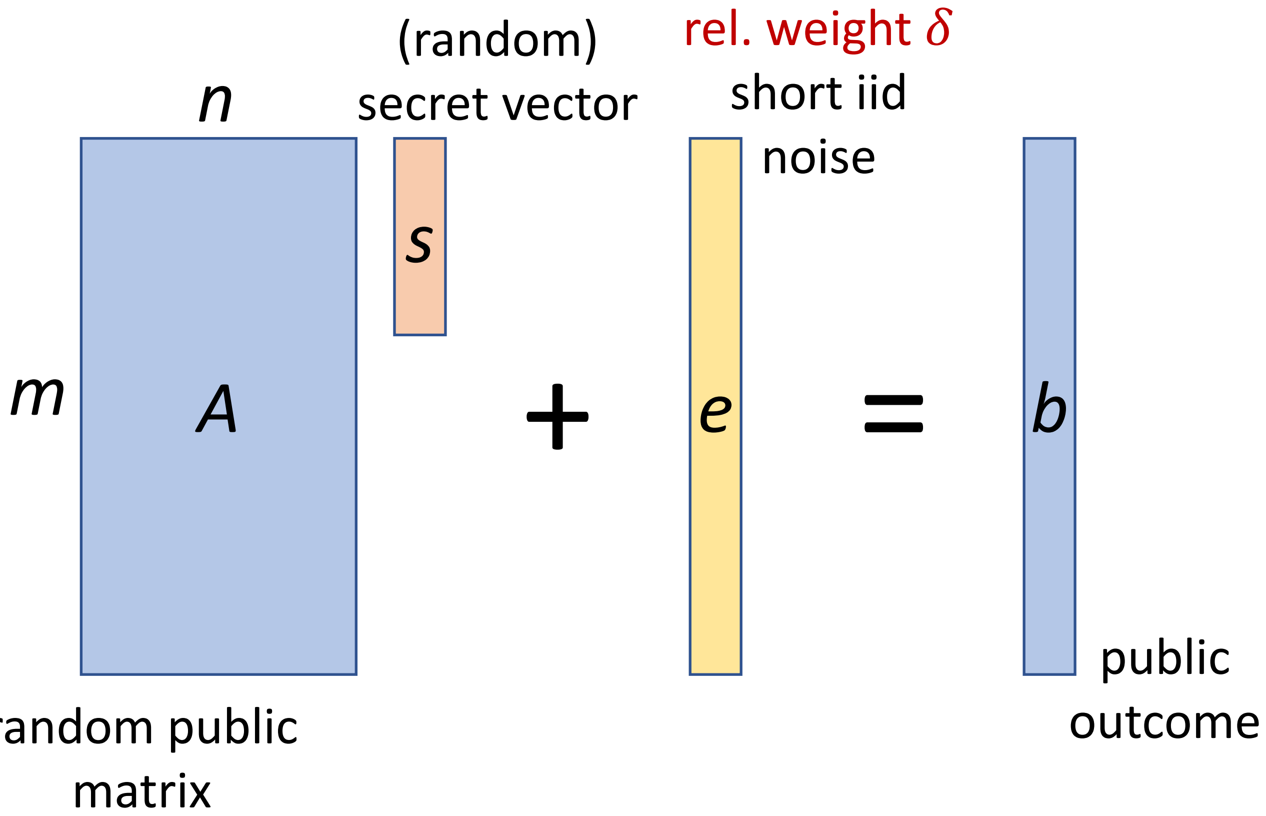
Learning Parity with Noise (LPN) [BFKL93]



Arithmetics modulo 2

Goal: $(A,b) \Rightarrow s$

Problem gets easy as noise gets sparse:

Solvable w.p. $e^{-\delta n}$ => poly. if $\delta = \frac{\log n}{n}$.

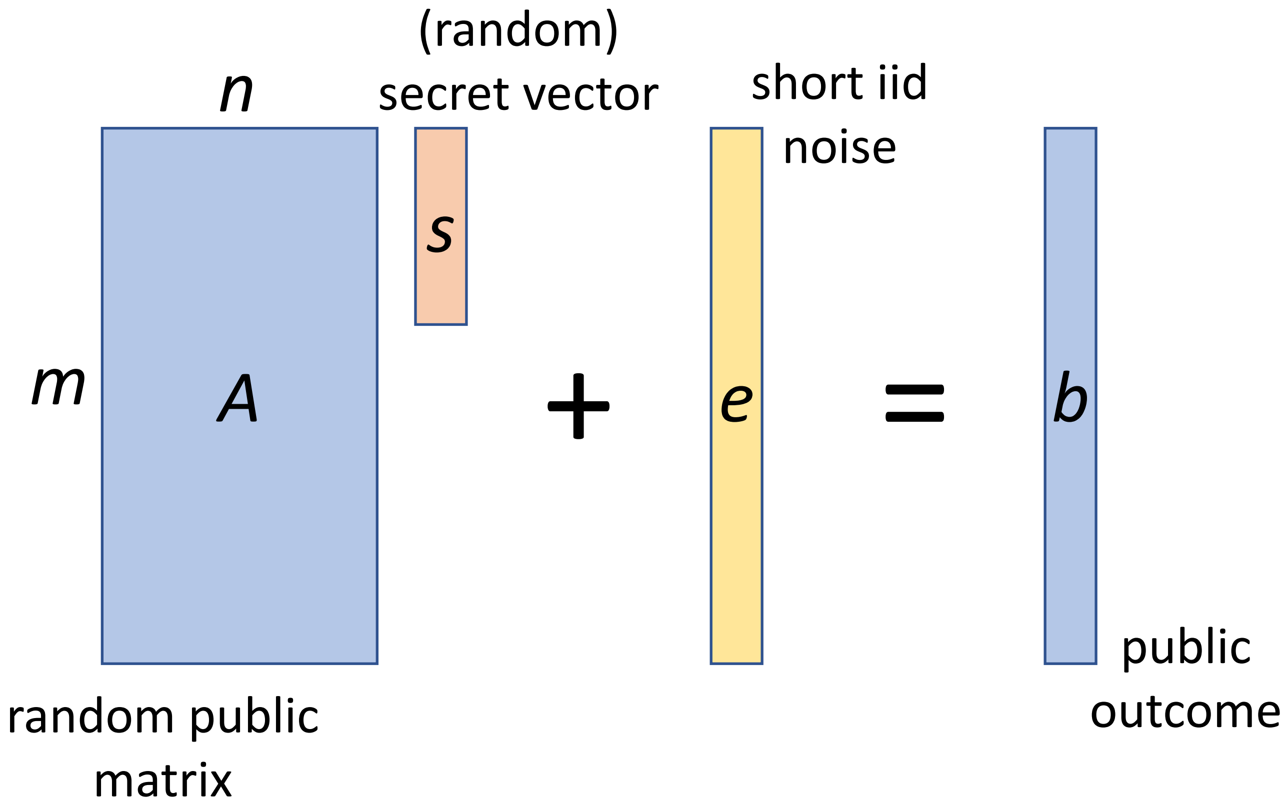
"Barely hard" LPN: $\delta = \frac{\log^2 n}{n}$.

Impossible w.p. $> (1 - 2\delta) \cdot m$

=> negl. if $\delta = \frac{1}{2} - n^{-\omega(1)}$.

"Super hard" LPN: $\delta = \frac{1}{2} - \frac{1}{\text{poly}(n)}$.

Learning with Errors (LWE) [R05]



Arithmetics modulo q>n, Gaussian noise

Goal: $(A,b) \Rightarrow s$

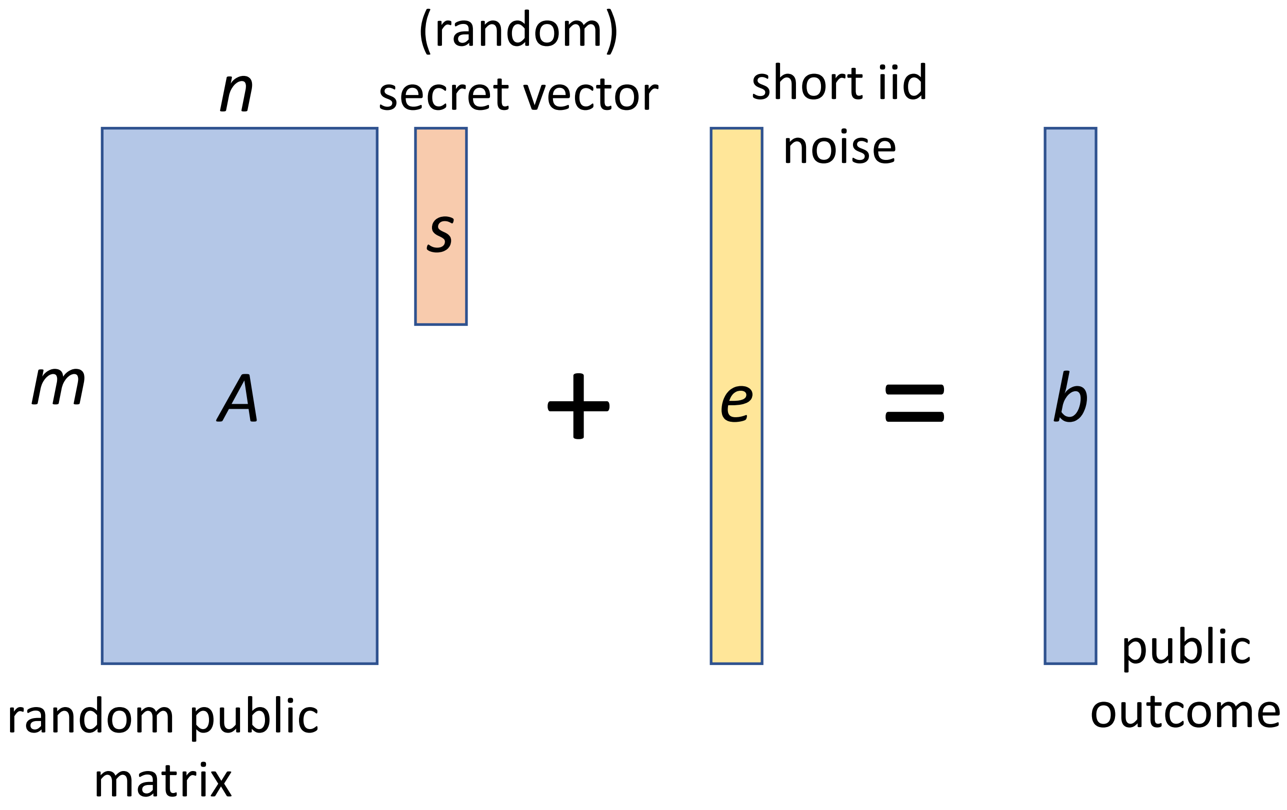
Known properties:

- Worst-case to average-case reduction.
- Contained in SZK (for useful params).

Known applications:

- Symmetric / Public-Key Encryption
- Collision resistant hash (CRH).
- Homomorphic Encryption.
- Attribute-Based Encryption.
- Non-Interactive Zero-Knowledge.

Learning Parity with Noise (LPN) [BFKL93]



Arithmetics modulo 2, Bernoulli noise

Goal: $(A,b) \Rightarrow s$

Known properties:

Known applications:

- Symmetric / Public-Key Encryption

Why so different?

Our Results

New properties:

- Worst-case to average-case reduction.

  LPN ≡ Average-case “Nearest Codeword Problem” (NCP).

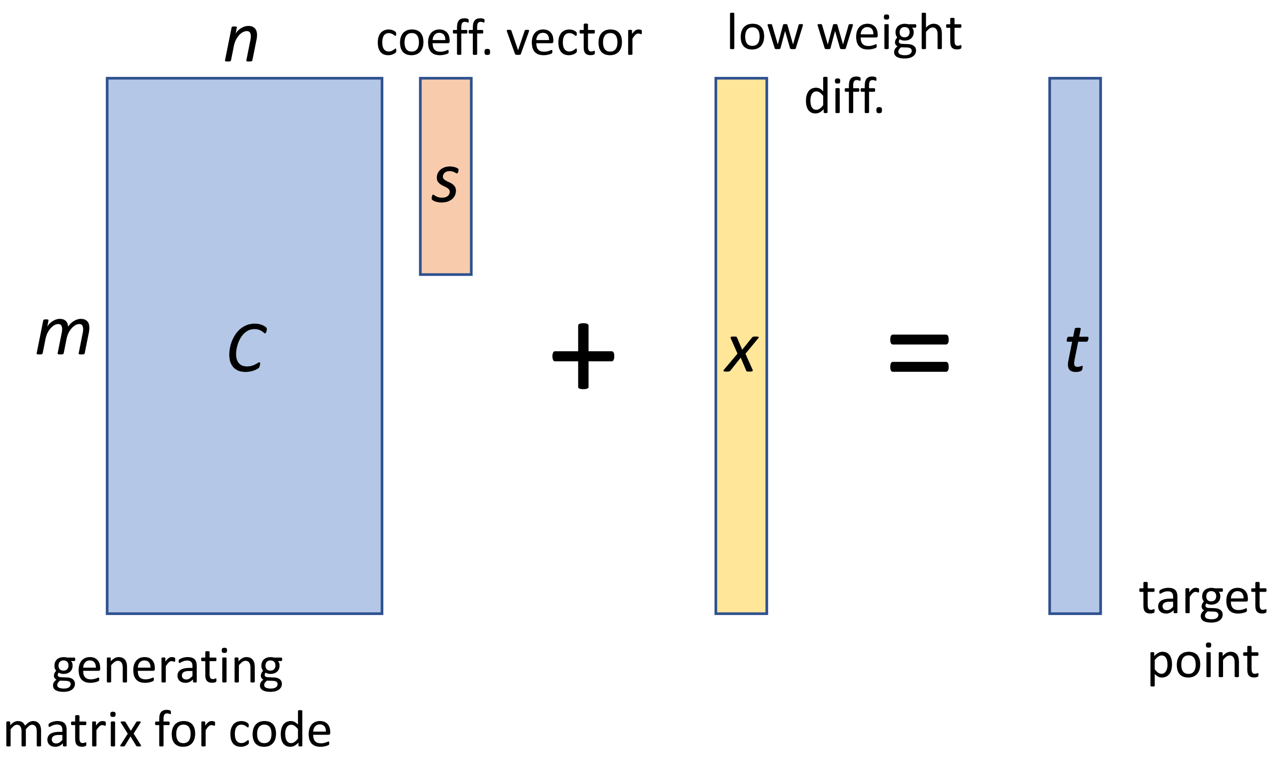
  We show: ”super hard” LPN is harder than “barely hard” (worst-case) NCP.

- “Barely hard” LPN/NCP contained in SZK.

New applications:

- Collision resistant hashing based on “barely hard” LPN (concurrently with [YZWGL17]).
- Follow-up works [BLSV18] extend to IBE, leakage resilience, KDM security (via laconic OT).

Nearest Codeword Problem (NCP)



Arithmetics modulo 2

Goal: (C,*t*) => *s*

Same as LPN except (*C*, *x*) are arbitrary.

NP-Hard in the worst case [ABSS93,DMS99].

We require *C* is *balanced*.

“Barely hard” and “super hard” regimes as in LPN.

Our Technique: Smoothing

[L05]: Solver for LPN with $m = n^{100}$, rel. weight δ'

=> Solver for LPN with $m = n^2$, rel. weight $\delta \ll \delta'$

Idea: Random matrix = *extractor.* Use to *rerandomize*.

Q: What is the min-weight to get entropy *n*? **A:** $\frac{n}{\log n}$ suffices!

The reduction: Generate n^{100} vectors *r*

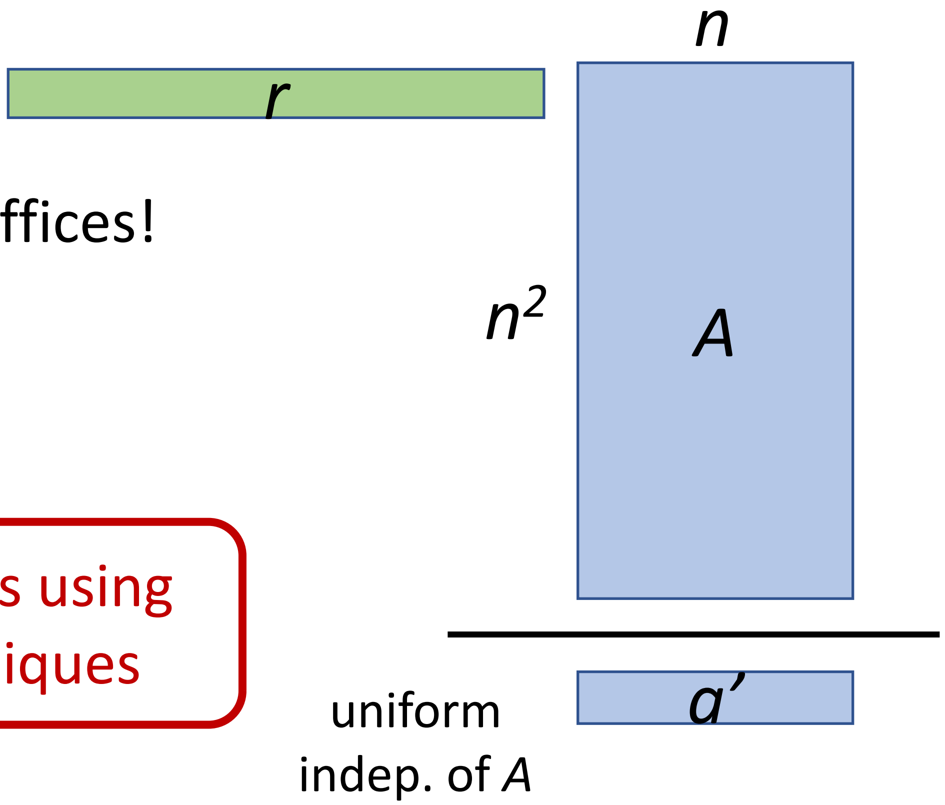
For each compute (*a'*,*b'*)=(*rA*, *rb*).

$\delta = \frac{\log^2 n}{n} \rightarrow \delta' = \frac{1}{2} - \frac{1}{poly(n)}$

CRH follows using std. techniques

Weight of … **gh**.

$1 - 2\delta' = (1 - 2\delta)^{n/\log n}$

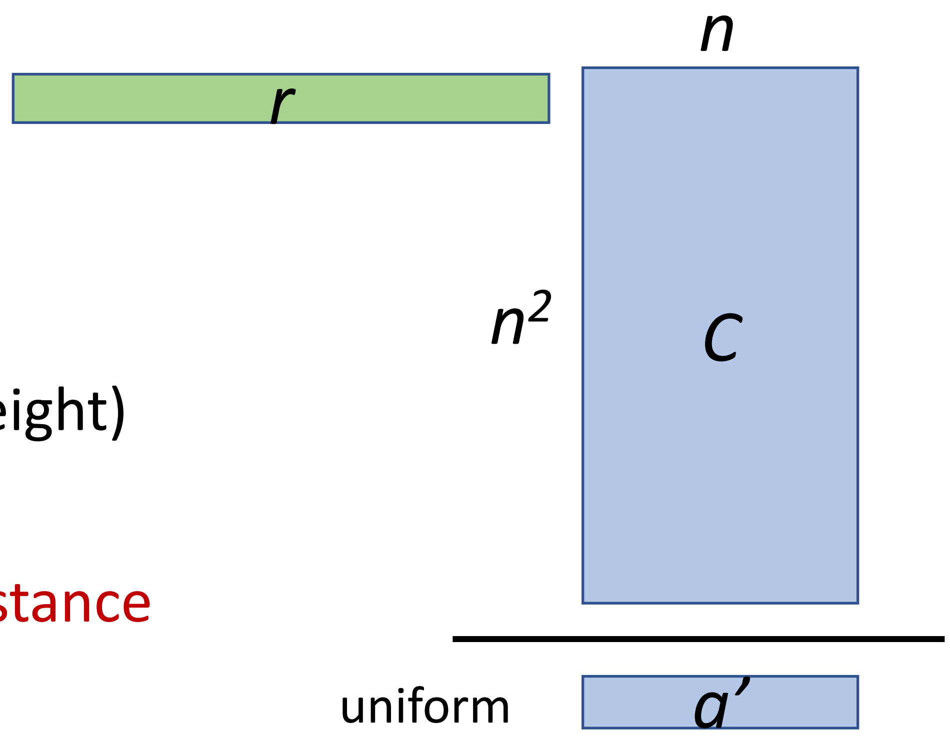


Our Technique: Smoothing

Apply technique to arbitrary (balanced) C ? Arbitrary C cannot be extractor.

Observation: Entropy extraction not needed, only extract from specific **smoothing** dist.

Can show this using harmonic analysis / linear distinguishers / Vazirani XOR lemma.



Note: We get $b' = rt = rCs+rx = a's+e'$

Need to argue that e' is indep. of a' (not just low weight)

Plug in barely hard NCP instance to get super hard LPN instance

=> Worst-case to average case reduction.

Connection to LWE / Lattices

Our technique is analogous to the concept of smoothing in the lattice world.

A distribution is smoothing for a lattice, if modulo the lattice it is uniform

<=> if its product with the dual bases is uniform (over cosets)

Usually in lattice literature: Smoothing using Discrete Gaussians,

in this work we extend the notion of smoothing beyond Gaussians.

Open Problems

Extend the params of our reduction.

Lower bound on smoothing? Non-trivial smoothing for unbalanced codes?

Is “barely hard” LPN/NCP actually not solvable in poly-time? Does balance help?

Construct more cryptography from LPN.

Vinod’s Question: Is there a CRH candidate that is provably not in SZK?

Thank you

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